

A LEVEL Cambridge Topical Past Papers

FURTHER MATHEMATICS MECHANICS P3

2020 — 2025

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1 - (9231/31_Summer_2020_Q6)

ANSWER

A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e . As a result of the impact, the direction of motion of P is turned through 90° .

(a) Show that $\tan^2 \alpha = \frac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e . [5]

2 - (9231/32_Summer_2020_Q6)

ANSWER

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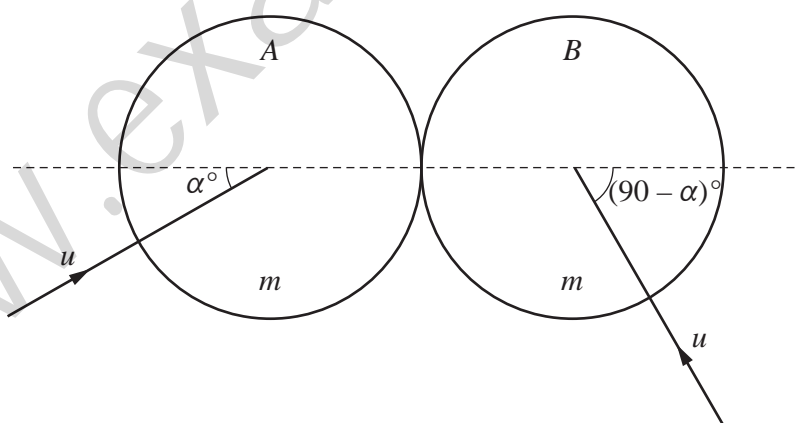
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(b) Find the value of α and the value of e . [5]

3 - (9231/33_Summer_2020_Q5)

ANSWER



Two uniform smooth spheres A and B of equal radii each have mass m . The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A 's direction of motion makes an angle of α° with the line of centres, and B 's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e .

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan \alpha = \frac{1+e}{1-e}$. [4]

- (b) Given that $\tan \alpha = 2$, find the speed of A after the collision. [4]

4 - (9231/31_Winter_2020_Q6) **ANSWER**

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B . The coefficient of restitution between the spheres is e .

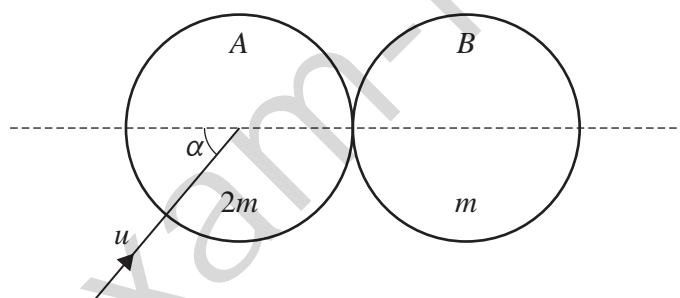
- (a) Find, in terms of u and e , the velocities of A and B after the collision. [3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B , where $\tan \theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B .

- (b) Find the possible values of e . [7]

5 - (9231/32_Winter_2020_Q2) **ANSWER**



Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B . Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan \alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

- Find the speed of A after the collision. [5]

6 - (9231/32_Winter_2020_Q4) **ANSWER**

A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r . The angle between the vertical and the normal reaction of the surface on P is θ .

- (a) Show that $\cos \theta = \frac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

- (b) Find x in terms of r . [4]

7 - (9231/33_Winter_2020_Q6)

ANSWER

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B . The coefficient of restitution between the spheres is e .

- (a) Find, in terms of u and e , the velocities of A and B after the collision. [3]

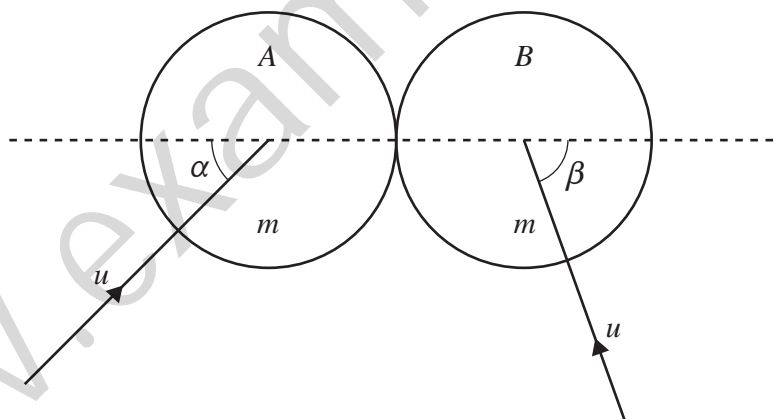
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The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B .

- (b) Find the possible values of e . [7]

8 - (9231/31_Summer_2021_Q6)

ANSWER



Two uniform smooth spheres A and B of equal radii each have mass m . The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A 's direction of motion makes an angle α with the line of centres, and B 's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2 \cos \beta = \cos \alpha$.

- (a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

- (b) Find the value of α . [4]

1 - (9231/31_Summer_2020_Q6)



(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)	
	$y = v \cos \alpha = eu \sin \alpha$	B1
	$x = v \sin \alpha = u \cos \alpha$	B1
	Divide: $\tan \alpha = \frac{1}{e \tan \alpha} : \tan^2 \alpha = \frac{1}{e}$.	B1
		3
(b)	$v^2 = \frac{1}{3}u^2$	B1
	$\left(\frac{u \cos \alpha}{\sin \alpha}\right)^2 = \frac{1}{3}u^2$	M1
	$(\tan \alpha)^2 = 3$	M1
	$\alpha = 60^\circ$	A1
	$e = \frac{1}{3}$	A1

	Alternative method for (b)	
	KE after impact = $\frac{1}{2}m(x^2 + y^2) = \frac{1}{2}m((u \cos \alpha)^2 + e^2 (u \sin \alpha)^2)$	M1
	From (a) $\sin \alpha = 1/\sqrt{(1+e)}$ and $\cos \alpha = \sqrt{e}/\sqrt{(1+e)}$	B1
	KE = $\frac{1}{2}mu^2 \left(\frac{e}{1+e} + \frac{e^2}{1+e} \right) = \frac{1}{2}mu^2 e$	A1
	This is equal to $\frac{1}{3} \times \frac{1}{2}mu^2$ so $e = \frac{1}{3}$	M1
	$\tan \alpha = \sqrt{3}$, $\alpha = 60^\circ$	A1
		5

2 - (9231/32_Summer_2020_Q6)



(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)	
	$y = v \cos \alpha = eu \sin \alpha$	B1
	$x = v \sin \alpha = u \cos \alpha$	B1
	Divide: $\tan \alpha = \frac{1}{e \tan \alpha}$: $\tan^2 \alpha = \frac{1}{e}$.	B1
		3

(b)	$v^2 = \frac{1}{3}u^2$	B1
	$\left(\frac{u \cos \alpha}{\sin \alpha}\right)^2 = \frac{1}{3}u^2$	M1
	$(\tan \alpha)^2 = 3$	M1
	$\alpha = 60^\circ$	A1
	$e = \frac{1}{3}$	A1
Alternative method for (b)		
	KE after impact = $\frac{1}{2}m(x^2 + y^2) = \frac{1}{2}m((u \cos \alpha)^2 + e^2(u \sin \alpha)^2)$	M1
	From (a) $\sin \alpha = 1/\sqrt{1+e}$ and $\cos \alpha = \sqrt{e}/\sqrt{1+e}$	B1
	KE = $\frac{1}{2}mu^2\left(\frac{e}{1+e} + \frac{e^2}{1+e}\right) = \frac{1}{2}mu^2e$	A1
	This is equal to $\frac{1}{3} \times \frac{1}{2}mu^2$ so $e = \frac{1}{3}$	M1
	$\tan \alpha = \sqrt{3}, \alpha = 60^\circ$	A1
		5

3 - (9231/33_Summer_2020_Q5)



(a)	Let w be speed of A along line of centres after collision	M1
	$\leftarrow mw = -mu \cos \alpha + mu \sin \alpha$	
	$w - 0 = e(u \cos \alpha + u \sin \alpha)$	M1
	Rearrange: $\sin \alpha (u - eu) = \cos \alpha (u + eu)$	M1
	$\tan \alpha = \frac{1+e}{1-e}$. AG	A1
	4	
(b)	$\tan \alpha = 2 \Rightarrow e = \frac{1}{3}$	B1
	$w = \frac{1}{3}u \left(\frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} \right) = \frac{u}{\sqrt{5}}$	M1
	Speed = $\sqrt{w^2 + (u \sin \alpha)^2}$	M1
	$= \sqrt{\frac{u^2}{5} + \frac{4u^2}{5}} = u$	A1
		4

4 - (9231/31_Winter_2020_O6)



(a)	$mu = mw + 2mv$	B1	Momentum equation (with m)
	$v - w = eu$	B1	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	B1	Both correct.
		3	
(b)	Perpendicular to plane: $y = ev \sin \theta$ Parallel to plane: $x = v \cos \theta$	B1	Both
	Speed of B = $\sqrt{x^2 + y^2} = \sqrt{v^2 \left(\left(\frac{4}{5} \right)^2 + \left(\frac{2}{3} \cdot \frac{3}{5} \right)^2 \right)}$ ($= \frac{2}{\sqrt{5}}v$)	M1	Speed of B
	KE of B = $\frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1	KE of B in terms of $u \cdot \frac{1}{2}$ and $2m$ needed
	KE of A = $\frac{1}{2} \cdot m \cdot \frac{u^2}{9} (1-2e)^2$ So $\frac{1}{2} \cdot m \cdot \frac{u^2}{9} (1-2e)^2 = \frac{5}{32} \cdot \frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1 A1	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	M1	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}, 1$	A1	Both values
	7		

5 - (9231/32_Winter_2020_Q2)



	Speeds v and w after collision $2mv + mw = 2mu \cos \alpha$	M1	Momentum equation with m . Correct masses, allow sin instead of cos
	$w - v = eu \cos \alpha$	M1	Restitution, with consistent signs
	$v = \frac{1}{3}u \cos \alpha (2 - e) = \frac{1}{3}u \cdot \frac{3}{5} \left(2 - \frac{1}{3}\right) = \frac{1}{3}u$	A1	
	Square of speed of A = $\left(\frac{1}{3}u\right)^2 + (u \sin \alpha)^2$	M1	Uses correct speed perpendicular to motion
	$= \left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2$ Speed = $\frac{13}{15}u$ (= 0.867u)	A1	
		5	

6 - (9231/32_Winter_2020_Q4)



(a)	$\uparrow N \cos \theta = mg$	B1	
	$\leftarrow N \sin \theta = mr \sin \theta \omega^2$	B1	
	$\cos \theta = \frac{mg}{N}$ so $\cos \theta = \frac{g}{\omega^2 r}$	B1	AG
		3	

(b)	$\cos \theta = \frac{r-x}{r} = \frac{g}{\omega^2 r}$	B1	Using trig of situation: must involve x
	In new situation: $r - 4x = r \times \frac{g}{4\omega^2 r}$	M1	Using new situation with $4x$ and 2ω seen
	$r - x = 4(r - 4x)$	M1	Combining
	$x = \frac{1}{5}r$	A1	
		4	

7 - (9231/33_Winter_2020_Q6)



(a)	$mu = mw + 2mv$	B1	Momentum equation (with m)
	$v - w = eu$	B1	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	B1	Both correct.
		3	

(b)	Perpendicular to plane: $y = ev \sin \theta$ Parallel to plane: $x = v \cos \theta$	B1	Both
	Speed of $B = \sqrt{x^2 + y^2} = \sqrt{v^2 \left(\left(\frac{4}{5} \right)^2 + \left(\frac{2}{3} \cdot \frac{3}{5} \right)^2 \right)}$ ($= \frac{2}{\sqrt{5}} v$)	M1	Speed of B
	KE of $B = \frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1	KE of B in terms of u , $\frac{1}{2}$ and $2m$ needed
	KE of $A = \frac{1}{2} \cdot m \cdot \frac{u^2}{9} (1-2e)^2$ So $\frac{1}{2} \cdot m \cdot \frac{u^2}{9} (1-2e)^2 = \frac{5}{32} \cdot \frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1 A1	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	M1	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}, 1$	A1	Both values
		7	

8 - (9231/31_Summer_2021_O6)



(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu \cos \alpha - mu \cos \beta$	M1	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos \beta + \cos \alpha)$	M1	Restitution.
	Both correct, masses seen.	A1	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	A1	AG.
		4	
(b)	$(v_2 = \frac{1}{2}u \cos \alpha = u \cos \beta)$ KE of B after collision is $\frac{1}{2}m(v_2^2 + (u \sin \beta)^2)$ KE of A after collision = $\frac{1}{2}m(u \sin \alpha)^2$	M1	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	M1	
	Simplify to equation in $\sin \alpha$	M1	
	$\sin \alpha = \frac{1}{\sqrt{2}}, \alpha = 45^\circ$	A1	
		4	